It's really the best way.
You first read the chapter.
You may have some remaining questions.
Then you attend the lecture.
Otherwise, you may not have a good understanding.
The MRI stuff is quite tricky.
You need to carefully consider concept, relationship,
and several times.
So review, preview, and reading again.
So all these are needed.
OK, so we talked about MRI imaging, the second part.
The first part, as you watched the video,
is about physical principle.
Now we explain how we can do MRI imaging based
on the physical model, the principle.
So far, so good.
So we have the medical physics, MRI physics,
as the first lecture, then the imaging.
And the next one, we talk about some different kinds
of MRI pulse sequences.
So you look into more detail in the applications.
So today, we first revisit what we learned.
The first part, a kind of review in case
you missed anything, and at least even you understand.
At least we can review a little bit.
So the idea is to model, to understand the model T1 and T2.
Those are very important parameters.
And then also the proton density.
Proton density is easy to understand.
You have a bunch of small magnets.
So overall, you have a magnetic Jason, the big M vector.
Then once you play with the M vector,
and then you have a T1 and T2 effect.
T1 and T2 are intrinsic to tissue type, and functional,
and even molecular imaging needs.
So you need to see how the signal will change
subject to T1 and T2, and how you can measure T1 and T2.
Then you can give multi-contrast MRI images.
So proton weighted, T1 weighted, T2 weighted,
you need to first know.
So this is more complicated than X-ray imaging.
So X-ray imaging, we have a linear attenuation coefficient.
Even the linear attenuation coefficient
is a function of energy.
You still have the mu that's talking about how many photons,
or what fraction of photons will be attenuated after X-ray beam
passes through certain thickness of a certain type of tissues.
But T1, T2, more complicated, so we
need to use some imagination, geometrical imagination.
Then the rest, the two-part slice selection,
and the sampling in k-space, really
we try to make MRI signals from the sample, from the patient,
or even more spatially specific.
We can map cross-sectional or volumetric image
of a patient.
So the second and the third part are really
emphasis of this lecture.
The last one, we talk about some MRI imaging principle.
So T1 and T2 relaxation, and kind of review.
So T1 and T2 are quite different.
T1, we talk about recovery of m vectors.
So along the z-axis, that's your main field direction.
And T1 time defined as 63% recovery
of our original m vector.

We call it m0.
So this is about spin lattice interaction.
So that will be something involved with the energy.
So you turn the m vector into certain direction,
like a 90-degree direction.
Then the energy you used, that you injected into the tissue,
will eventually be lost due to spin lattice interaction.
And it will gradually return to the steady state.
So that is m vector ought to be aligned along the main field
direction, the direction of b0 field or the z direction.
So on the other hand, the T2 parameter,
talking about defacing effect.
So the m vector is not an individual thing.
It's really a collective phenomenon.
So all of the small magnets, really spinning protons,
these are very small magnets.
So altogether, they form big m vector.
You flip the m vector, say, 90 degrees.
So you flip the m vector along z direction.
90 degrees maybe along y or x direction.
So this big m vector really consists
of many small magnets, so all pointing to y direction,
for example.
But at a given time, those small magnets, some spin faster,
some spin slower due to, say, inhomogeneity in the b0 field
or the local field.
And the local field, there are ions, some molecules, motions.
And all these things contribute to inhomogeneity
of the magnetic field.
So the small magnets, so you see those small magnets,
originally, they all line up along the x-axis in this way.
If you apply a 90 degree pulse with respect
to the y-axis, then you flip the m vector along the x-axis.
But at a given time, so this will deface.
So some positional motion, faster,
some delayed a little bit.
And at a given long enough time, so all these small magnets
or spinning protons pointing to different directions.
So the overall field is no longer this m0,
but it becomes 0.
So the decay shown modeled as exponential curve,
something like this.
So this is a recovery.
This is a decay.
So this is just the t1, t2 thing.
And let me give you a little bit more mathematical detail
so you understand why the recovery and the defacing
all follow the exponential curve.
Certainly, one is growing, the other is decaying.
So it's something like this.
So the change rate, either growth or decay,
is described by this ordinary first order
differential equation.
So first derivative, the change rate
is proportional to the original quantity.
You're talking about change of y.
Then the change rate is proportional to y.
So you keep seeing this kind of first order differential
equation in multiple imaging modalities,
like x-ray imaging, so you have initial beam intensity.
It will be attenuated.
The amount of attenuation, the change of the x-ray flux
is proportional to the initial flux, i0.
So likewise, here is y, and then we

talk about initial m vector, m0.
So you've got this relationship.
Whenever you have governing relationship like this,
so you can go through mathematical steps.
So first derivative like this, you just
rearrange it a little bit, so y dy equal to k dt.
Then just do integral on both the other side of the equation.
So you've got a solution.
The solution is shown something like this.
Then that indicates that whenever the process is
described by the first order differential equation,
the solution must be something shown here.
y must be either exponential growth or decay,
so constant multiplied by an exponential function.
So with index kt, t is time here.
So you'll see why you have exponential form of decay,
or growth, or attenuation in earlier imaging
modality, x-ray imaging.
So you understand why you have that.
And for t2, and the t2 really we need to look into more detail.
So t2 has two components, and the t2 is defacing.
The first defacing we call it t2 effect spin-spin interaction,
because all these spinning protons,
they close each other.
You do not have absolutely uniform distribution.
So if you look microscopically, so the spins,
they interact together.
And those are fluctuation in local magnetic field.
So we call these red components pure t2 effect
due to spin-spin interaction.
The physiological behavior in the soft tissue,
you cannot do much about it.
So that's just a biological thing.
The inhomogeneity magnetic field due to the spins
inside the tissue.
On the other hand, you subject the patient or small animal
in external field, the B0 field.
We say B0 field is uniform.
That's just an idealization.
In fact, the B0 field cannot be absolutely homogeneous.
So there will be some inhomogeneity inside.
So this inhomogeneity is kind of fixed.
You make up the B0 field of a given scale.
The field is not uniform.
Maybe certain part of the field is slightly weaker.
The other part of the field is slightly stronger.
This is fixed.
So you have constant inhomogeneity magnetic field.
You have some field weakness here and a little strong here.
In the weaker field, the local spins
will do precision a little bit slower.
And in the stronger field, all related to B0 field,
the spinning will be a little bit faster.
So some slow, some faster.
So this contributes to the defacing effect.
That's a T2 effect.
So both biological inhomogeneity in magnetic field
and the physical external field inhomogeneity
all contribute to the same thing.
But they come from different reasons.
External reason is fixed.
Later on, I will explain.
You can correct for that.
But for biological spin and spin-spin interaction,

you cannot do much.
So both factor together, and we call it T2 star.
This external field inhomogeneity effect
is denoted by T2 plus.
So T2 star is due to two effects.
External field inhomogeneity and the biological spin-spin
interaction, the two things together.
And we say this is a combined effect.
But you have a reciprocal relationship.
Why is that?
You see this relationship?
So you have this K. So the K is due to spin-spin interaction.
The defacing is proportional to spin-spin interaction.
This is the one thing.
You can call it a K1.
And also, the defacing effect could
come from an external field inhomogeneity.
And you have another equation.
You'll call it a T2.
So really change rate, just a small time instant.
Due to K1 and K2, you think here is a K1 and a K2.
So then the time constant is a reciprocal of the K1 and the K2.
So you have this relationship.
So if you still feel not totally clear,
let me just review with you what we learned in nuclear imaging
modality.
So when you learn each modality, you
should think about other imaging modality.
And the things are well-connected
for medical imaging.
So when we say nuclear decay, so you call it gamma rays.
And you inject a nuclear tracer into human body.
And the tracer will subject to two kinds of decays.
One is physical radioactive decay.
This happens all the time.
And also, when you have a tracer injected into bloodstream,
the tracer will eventually be cleaned out
through biological circulation.
Mainly, you pass water.
Then the radio tracer will go out of the body.
So you have two mechanisms to get rid of radioactive tracer.
So one is a biological somehow like a spin-spin interaction
due to physiological molecular motion.
The other is circulation, something like an external field.
In this case, you have a very same reciprocal relationship.
As you see, this tau, tau is the coefficient k before.
So that is, see here, is t.
t is reciprocal of k.
And here is time.
Time is, say, this lambda plays a role of k
in the differential equation in the nuclear imaging modality.
So reciprocal of lambda give you half life of tau.
So then you have this reciprocal summation.
The summation really means lambda 1 plus lambda 2.
Lambda 1 is biological decay.
Lambda 2 is biological decay, just the urine circulation.
So you have the change rate defined
by the first order differential equation.
The coefficient is lambda 1 plus lambda 2.
Then the individual time constant
is reciprocal for lambda 1 and lambda 2, respectively.
So you have this overall.
This is basically say overall lambda
equal to lambda 1 and lambda 2.

And the lambda 1 reciprocal is half life.
So forget about the constant.
The constant is there because you talk about half life.
Then you have log 2.
So this is the idea.
So do not be confused by, say, this is equation 2.4.
And then the other equation is 4.29.
So these two equations, mathematically the same thing.
So if you do not follow me, so just
need to think a little bit about.
We talked about T1, T2.
And the T2, really, we talk about three things.
And the biological T2, just a pure T2.
And the T2 plus, T2 plus is due to external field
in homogeneity.
That contribute to the facing effect.
And then both combined, you have T2 star.
You have T2 and T1.
And how do you measure these parameters?
So for T1 measurement, we can use inversion recovery.
And the next slide, I will show you graphically.
It's just a kind of review with you.
So first, I have a 180 degree pulse.
So I'm right here.
You basically flip upside down.
You still do not have transverse components.
Only transverse components will introduce alternating field
from the sample.
You put a nearby coil, you will detect a signal.
So without any transverse components,
you'll have a coil close to sample or patient.
You wouldn't be able to measure any signal.
So once you apply 180 degree pulse to the m vector,
all of a sudden, it becomes upside down.
So no transverse components in this case.
So if you put a coil close to patient, no signal at all.
So you need a transverse component.
After time delay tau, then you have a 90 degree pulse.
You introduce a transverse component.
Then you can measure signal.
So this is the signal.
The signal also decay here.
So you can think about this decay.
Why you have this decay?
Transverse signal keep doing things like this.
But this transverse signal will subject to decay,
get smaller and smaller due to dephasing.
So this dephasing is due to, you can ask yourself,
due to T2 or T2 star?
It's due to T2 star.
Because it's just that both field inhomogeneity
and the spin-spin interaction are there.
So you measure this signal.
What is the amplitude?
That depends on tau.
When you will flip 90 degree.
So this is illustrated here.
Review with you is so important.
I'll see.
You have 180 degree.
So the M0 becomes minus M0.
So if you don't do anything, so this is negative.
So originally, you have a positive M0.
All of a sudden, become negative M0.

So this negative M0 will gradually
recover to the original M0.
Just return to the normal, the stable status.
If you don't do anything, just see this negative taper point.
Gradually moving up at some point becomes 0.
Then give more time.
It will eventually return to M0.
Nothing happened.
So it depends on the tau.
So this gets smaller and smaller.
That becomes positive.
It becomes greater and greater.
So this amplitude initially becomes smaller and smaller
until 0.
Then grow back towards M0 again.
And it depends on tau.
Say at a certain time, if it grows to this way.
So they got here.
Not reaching M0 yet.
At this point, you apply a 90 degree pulse.
Flip it along y-axis.
You have a transverse component.
At that time, you can do measurement.
At this time, you can do measurement.
So see this is subject to T2 start decay.
So you have this free induction decay.
So you have a signal measured.
You can do so multiple times.
You can do so multiple times for different tau.
And you see the relationship between the signal you measured
and as a function of tau n.
Then you can plot the relationship.
You can find the negative slope.
And how to do this?
You need to think a little bit.
For tau n equal to 0, you can measure this immediately.
So this is known.
When tau equal to 0, you immediately measure this M0.
You arrange a little bit.
On the left-hand side, you have all unknown.
Right-hand side, you have a exponential function.
But it's with an index minus tau n divided by T1.
You do not know T1.
But if you do log for this exponential function,
this T1 will come out.
So you see the straight line with a slope minus T2.
So you can minus T1.
In this way, you can find the T1.
So think about this.
So this sentence is not exactly right.
If you directly do the log, you cannot get this negative T1
slope.
You need to do some arrangement.
But remember, this M0 is known.
Because you can measure this M0 as noted here.
So just remember that.
OK.
How do we measure T2?
Spin echo is probably, I think, the most important, most
representative pulse sequence.
So the spin echo is a very clever design.
And the data is used to measure T2.
And if you do not do spin echo, the signal decay
will subject to T2 star.

And what we want to measure is T2.
Because T2, pure T2, is a biological effect.
The T2 star, largely due to external field.
So your physiological, pathological status
has nothing to do with external field.
So you really want to measure this T2.
How do you measure T2?
So spin echo.
So you see, you first light the signal,
deface a little bit.
Then you flip it 180 degrees.
So this part, you see, this is delay, the phase delay.
This is phase, advanced phase.
After 180 degree flipping, this delayed one
will continuously delay.
These are fast moving spins, continuously moving fast.
Then, given the same time, they will refocus at this point.
So this is a very smart design.
Another trick, you can do the same thing.
So you have the m vector, flip it by 90 degrees.
So you have transverse components.
And these overall transverse components
will subject to defacing.
So some are slower, so in red.
Some are faster, so in black.
Then, at this time, you flip with respect to these axes.
You flip it 180 degrees, instantaneously.
So red is moved here.
And the black takes the original position of red.
And we know the red is slow.
And you put it ahead of time.
It is still slow.
The black is faster.
You put it behind, it is still faster.
Given the same amount of time, the black
will realign with these, say, y-axis.
And the red will be gradually delayed to the same position.
At that moment, all the components relined again.
And because you have all these small components
aligned along the same direction,
so you have the maximum amplitude overall.
And this amplitude induces MRI signals
in nearby detecting coils.
So we have a spin echo signal there.
And an even simpler explanation, you
have multiple students trying to race.
Starting point, they all find just the start line.
After a signal, they all run to the finish line.
Some are faster, and some slower.
So for a given time, they cover different manages.
And at that time, we instruct them, keep running,
but in opposite direction.
So the faster guy has a longer distance to cover.
The slow guy has the shortest distance,
and the slowest speed to cover.
Same amount of time, they will all come back
to the starting line.
So they realign again.
Just the idea like this.
So this spin echo idea.
With that, you can get T2, pure T2 measured.
So very clever design.
So we have two parameters for you to know.
So one is time for echo.

So the time for echo.
So basically, from 90 degree pulse to 180 degree pulse,
and you take a time being equal to Te divided by 2.
OK, then given same amount of time, another Te divided by 2,
you have this spin echo.
The spin echo amplitude really reflects the decay
due to pure T2.
Nothing to do with external field inhomogeneity.
So this is the first parameter, time echo, so Te.
And then the whole thing, you wait for time for repetition.
You can do the same trick again.
Just do 90 degree, 180 degree, then you
expect another spin echo.
So at this time, the T time repetition
is expected to be long enough so that the original m0 vector is
already assumed.
So you just have the whole system return to the steady state.
So this is a normal case.
If you make a TR 2,000, then the system
doesn't have enough time to return
to its stable status.
So along g-axis, you do not have the original m0 vector,
maybe a little bit smaller.
So this is significance of Te and the Tr.
So by definition, you see Te should be less than Tr, right?
So something like this.
And this part involves imaging.
I will explain the imaging from the next slide up.
So right now, you don't need to pay attention to this part.
And you will see this slide again.
But at the top part, say spin echo for the whole sample.
The bottom part really to get the spin echo measurement
for each individual pixels.
And at the top part for whole sample,
and then you get the overall information
about the patient or sample as a whole.
But how you can resolve the information?
The T2, T1 is not for whole sample.
You really want to see the sample as a collection
of pixels or pixels.
You want to know T1, T2 for each individual pixel.
How you do that?
Then we move to the next part.
We want to be spatially specific.
We want to get a signal from a particular slice.
Not every wire, just one slice.
And not only from that slice.
And we want to know where in the slice we get the signal.
So you need to encode each pixel to get a signal a little bit
different from each signal.
So you can finally measure T1, T2
for each and every pixels or voxels.
So first part, we need to just find a slice we want to image.
So this is about precision of frequency on B.
And in the textbook, I think we mentioned
this in previous lecture.
So you can go through the derivation
if you have a problem and just review the previous lecture
or ask me or TA.
But the conclusion goes through all these steps
that you will understand.
So the precision of frequency is proportional
to the local field.
So overall field, the stronger field, the m vector

or the small spinning top will do precision.
It's also the B. Anyway, I will correct it later.
So the stronger field, it will move faster.
So you just guide this picture.
So last line is clear.
The angular frequency, precision of angular frequency,
it's proportional to local field.
So it's B0 then proportional to B0.
Later on, you change B0 to B prime.
Then the omega is proportional to B prime.
And then you have a coefficient.
So this is just the physical constant gamma.
And if you want to know more, then you
can just check the derivation for this mechanical,
precision of motion.
And the angular frequency in this case
is proportional to the gravitational force.
And I put it in green.
So if you like, you read.
Otherwise, do not worry about it.
Then the great idea is to introduce a gradient field
on top of the background field B0.
So the idea like this was a Nobel Prize.
So this is something very important to do encoding.
We know the precision of frequency
will be the same in the B0 field.
But after we apply a gradient field,
we just modify the field so that the larger Z coordinate
and the stronger the field, so the precision of frequency
will not be the same.
Really depends on the frequency.
It will be still the original B0 specified frequency,
only at Z location, Z equal to 0.
So this is a field gradient.
And once we have a field gradient,
you can apply external rotating magnetic field
to push the M vector at the resonant frequency,
precision of frequency.
And in the previous lecture, remember,
photograph a lady keep pushing a rotating door.
That same idea.
And even earlier, we watched a small video tape.
And the instructor explained how MRI will work in less than 10
minutes.
He said you shoot RF frequency signal.
That's a rotating magnetic, because RF signal, RF pulse,
is electromagnetic field, is oscillating field.
And you send the RF frequency in resonance
to the precision of frequency.
You keep pushing local spinning tops in a certain direction.
Then you can just flip.
Again, you have a precision motion.
You flip the M vector at that frequency
towards one of the horizontal axes,
like X or Y axes.
So you send this rotating field within this frequency range.
So the frequency range will be from the central frequency
omega s plus or minus delta omega s.
So this whole field, you send the signal
in this frequency range.
So in time domain, this is a rectangular function.
Then the inverse Fourier transform
gives you a synchro function, shown here.
So even in time domain, you send the RF signal

in the form of a synchro function.
In frequency domain, you have this rectangular function.
This rectangular function will be in resident with spins
within a certain frequency range.
So the precision of frequency range
depends on location, because we introduced the gridding field.
The gridding field is different depending on z.
So for certain ranges, along z, you have a frequency range.
And we match this frequency range with this range.
And that is implemented by this synchro-shaped RF signal.
So you send that one.
You are in resident with that range.
So this range is really a slice thickness.
So you send the RF signal.
Only those spins inside this z range,
inside this, within this slice, will be flipped.
And therefore, spins outside this range,
and you do not have any signal, because this frequency range
is only defined within this frequency range corresponding
to the slice thickness.
So graphically, this slice thickness
is determined by this frequency range
due to this gridding field.
So you have selectivity.
So you only select that slice.
So I explained the idea for X-slice selection.
And in your textbook, you have this figure 4.16.
And you can do slice selection using x-gradients,
y-gradients, z-gradients.
So actually, you can select the slice along any orientation,
not only along x, y, z, and even opposite directions.
So this is flexibility we have with MRI imaging.
Any slice, you can make the selection.
So this is something we need to be specific about MRI imaging.
And before applying gridding field,
and we could only get T1, T2 signal from a whole sample,
but now we can be so specific.
We only collect signals from this slice or that slice.
And any signals outside the slice,
the spins here, here, here, will not contribute to the signal.
Because any spins, like here, here, not in the slice,
and we didn't play with those external spins
because we have this resonance signal only
defined in this range.
And this frequency range happened
to be defined by this slice definition.
So this is slice selection.
And there is a technical subtleties
about the slice selection.
So we applied a gridding field.
By definition, the gridding field is not homogeneous.
It's linearly changing.
And any external field deviated from a homogeneous field
will contribute to defacing problem.
And the same comment applies to a gridding field.
The gridding field and the linear gridding.
So right-hand side, the field is a little stronger.
The left-hand side, the field is weaker.
So stronger field corresponding to faster,
precise motion.
And the weaker field, on the other hand, moves slower.
So you put all the signals together.
And this defacing effect will reduce the signal
substantially because they do not move in sync.

So this is a problem.
So you need a refacing gridding.
You still select the slice.
Within the slice, you have a defacing
due to the external field.
Then again, you use the opposite field.
Just to make sure, within this slice,
all defaced spins will be brought back.
So this is described by equation 4.41 in your book.
So this is the defacing effect.
So phase factor is proportional to z.
And this coefficient, z, z.
So you apply the slice selection pulse over time tau.
But here, the factor is only tau over 2.
Why is it tau over 2?
So this is our original spin.
Within time tau, it will be defaced to this position.
And for the whole slice, some spins goes this way.
The other spins goes the other way.
The middle position, so it's a whole range for tau.
The middle position is defined by time tau over 2.
So you just think everything is defaced.
But the whole group, you take a middle one as a standard.
And that corresponding to the time vector is not 0.
It's not tau.
Rather, tau divided by 2.
So you really correct with respect
to the average of all the affected spins.
So to correct for this, then you introduce this tau refacing.
Then you have a z gradient introduced opposite
to our original zz.
So this refacing gradient, gz times this tau refacing.
So overall effect should be equal to tau divided by 2 times gz.
So this is the balancing equation.
So on one hand, the original, this first equation,
you do defacing.
The overall effect, some defacing, some less.
So the average is shown, graphically
shown as this black arrow.
So some with respect to this average,
some spins moves faster, some spins moves slower.
So after time original slice selection, time tau.
So you got this picture.
Then we apply in here, say, in opposite polarity,
you got a compensating gradient.
So g refacing, g refacing.
And then you have the time tau refacing.
This overall effect, how strong?
How strong is the field?
How long time you let the field last?
So this product, multiplication,
give you total phase change.
So you change the phase in one direction,
then you bring it back.
So this is a balancing equation.
So after that, you select the slice.
Yet the linear gradient induced defacing
has been canceled out.
So all you have is a signal.
So within the slice, you have a signal.
How strong is the signal?
The signal is proportional to the total number
of spins inside that slice.
So you have a double integral.

It just need one slice, a double integral
for the selected slice.
For the selected slice, you have the xy coordinate system.
So you just say the signal strength within this slice
is proportional to the total proton density.
So if you just measure the signal, no T1, T2 yet.
It's just the proton density overall integrated.
If you don't select the slice, the signal
you have, like what we mentioned,
how you measure T1, T2, that will be triple integral.
So rho, x, y, z, so all the small spinning protons
inside the volume will be selected.
Now we already made a great step forward.
Instead of triple integral, we just removed one dimension.
And we only got signal from the selected slice.
So this is a big step forward, but not good enough.
Because we really want to have individual, say rho, x1, y1,
rho, x2, y2, individual point we want to know signal.
So what is the individual pixel value for proton density
and for T1 specific to that pixel location?
So that will be covered in the next part.
We say sampling in case-based, this
is the most important thing.
And we will see the case-based theorem
is formulated in terms of Fourier transformation.
The MRI signal with the gradient encoding, phase encoding,
and the frequency encoding will naturally
give you Fourier transformation.
And this is, I would say, quite tricky.
So you need to review, read the green textbook.
If you just read the textbook carefully enough,
you should be able to understand.
The book is well written.
And if you're still not clear, then you
review the lecture I gave.
And you review, you keep thinking again and again,
maybe three times you will understand.
If you didn't read anything yet, then you
may not be able to follow the lecture.
That's a very natural thing, I would say.
And everyone, including myself, I
studied multiple times before I really
see why you have a case-based or case theorem.
The case theorem is equivalent to Fourier slice theorem
for X-ray computed tomography.
So the third part, sampling in case-based,
will make the final step forward.
Instead of getting collective signal from all the protons
in the slice selected, we can resolve individual locations.
We know what's the row density for a given pixel location.
What is T1 in that pixel?
What is T2 in that pixel?
So once you know all this information, pixel-wise,
you have a cross-sectional image.
That is the purpose of tomographic imaging.
So let's have 10 minutes rest.
Then we enjoy the case-based theorem,
how we understand heuristically and mathematically
how to do MRI imaging tomographically.
OK.
So here is the equation.
We just explained.
And this is a puzzle, actually.
And if you are motivated enough, and before reading further

or listening to my lecture, you can think what you will do.
So if you see this equation, and you see you can select it,
you can single out one slice.
So signal only comes from this slice.
So this is better than, this is more specific,
than whole volume or whole pixel sensing.
And now you only get signal from one slice.
How from this equation?
And then you use some trick to resolve
the spatial information.
You know, say, row T1, T2 at a given xy, say, x0, y0.
For that particular location, how can be more specific?
And this is something I would suggest
you think hard, what you will do.
And in this third section, we explain a way
through phase encoding and frequency encoding.
And then you can follow.
The chance is that you understand,
then you see, OK, this is a great way.
And also, you can think independently.
In addition to the way I explained to you,
do you see any other way you can do the same thing?
And I can assure you, there are multiple ways
to do the same thing.
And what I'm going to explain to you
is very neat and highly efficient.
And the other ways may not be as efficient as what
I'm going to explain to you.
But still, you can think out of box.
So if you have this problem, you want
to resolve cross-sectional image,
tomographically, what you will do.
But anyway, so now it's time for us to explain.
And we see all the signal coming from the selected slides.
And you have the signal.
You have no information where the signal comes from.
You know it's from the slides.
But within the slides, where the signal, how signal
contribute individually towards the overall signal,
you want to make a sparsely resolved imaging.
So how you do that?
So phase encoding, frequency encoding,
are very heuristic ways to tell you
how you can differentiate the signals from the slides.
If you do not do anything, all signal
come out in same frequency, added together.
So you just do not know what are the spatial composition.
But with some further processing,
as shown here, you can add a gradient encoding again
along this direction.
So this encoding lasts for a certain amount of time.
What will happen?
So initially, all the signals, see, is in phase.
But after a certain time, with the phase gradients on,
and the gradients may be zero.
So relative phase is the same at this step.
But in a little bit lower location,
the field gets, say, a little bit stronger.
Then you have a phase factor proportional
to the local field stress.
So you gain a phase factor.
Then you move a little bit further down.
The phase factor will be even larger,
because this is a linear gradient.

The gradient gets larger and larger.
And the result is that the precision of motion
gets faster and faster for a given time.
So the higher frequency, higher precision of frequency,
will move larger, cover a larger phase range.
So all these will be summarized into total phase change.
The zero phase change means the gradient field has no effect.
There is zero value there.
The larger gradient field has higher
precision of angular frequency.
Then larger phase factor gets accumulated.
So larger is even larger, furthermore.
So this is the largest phase delay.
So if you don't do anything, you already make some difference.
So any pixel along this line, no phase change.
Along this line, you have 180 degree phase factor
accumulated.
So you know, now if you collect the signal again,
all signal come out.
But those with smallest phase delay or no phase change
must come from top.
And those with larger phase delay
must come from the bottom of this particular slice
you just selected.
So this way can help you resolve spatial information.
It's not totally clear yet.
You only can tell from which row.
Like a student, I know some voice
come from first row, second row, last row.
But I don't know which one yet.
Then we come to the frequency encoding here.
So phase encoding idea, just like what I explained.
And if you read your book chapter,
it explains that the phase factor is
due to local precision of angular frequency.
And the total time applied for PE, phase encoding.
So this is a product.
And the omega is proportional to the local field.
Local field is not all the same.
It's linearly changing, because the gradient field
for phase encoding with this gradient gy.
So gy times y is linearly changing.
So gy times y gives you local field change at coordinate y.
So you got this thing.
So this is phase factor accumulated.
As I explained, if you collect the signal now,
you still have signal from the same slides.
But they will not be with the same phase factor.
Phase factor is a function of y due to the phase encoding
gradient field.
So you got a little bit more information.
So you got things like this.
And then now you have line-wise resolved information.
But you want to do tomographic imaging.
You want to spatially resolve pixel-wise, point-wise,
specific information.
Then the next idea comes out.
It's called frequency encoding.
So after phase encoding, you do not read any signal.
Just know that the information already phase-encoded.
Then you apply frequency encoding gradients.
This is gs, the frequency encoding.
So see the frequency encoding like this.
With this encoding, you start recording signal.

So what will happen?
The local field here is high because of the gradient field.
So gradient field equal to normal price.
So this is a very intelligent way to do the imaging.
So at this higher gradient field due to this gs,
the precision of frequency will be highest.
So if you read a signal out and some signal
at a very high frequency, you know the signal must
be coming from this column.
And those signals come out with lower precision of frequency
must be due to relatively weaker field.
So now that the field is no longer homogeneous
over the whole plane, but rather you have a gradient field
and the faster oscillation, you immediately
have a hint the signal must be from right-hand side.
If the signal come out with a little bit of slow frequency,
you know it must be from left-hand side.
All the low-frequency signals you know from left-hand side,
then you can look at more detail.
You check the phase angle.
The phase angle, you see, the smaller phase angle from top,
the larger phase angle from bottom.
So with phase encoding and the frequency encoding all together,
and you can do point-wise information collection.
So this is just the rough idea.
Why is it two more gradient fields?
Initially, we use phase selection gradient, say,
along z direction.
Then you do y direction encoding, x direction encoding
for phase and the frequency encoding, respectively.
So you gradually eliminate one dimensionality at a time.
And eventually, you just make a 3D-specific localization.
You get information from individual points.
And this is the key idea for topographic imaging.
So mathematically, so let's just say
you still have signal from the whole slice.
But with phase encoding and the frequency encoding,
you can get this last equation.
And the first equation is just say without any phase encoding.
You just have the frequency encoding.
What do you have?
So you have this part of the signal.
If both together, then you have something shown here.
And looking at this equation, pretty much
like a Fourier transformation.
So this is a function.
And you have x components and y components.
And we will see more details later in this class.
So the frequency and the phase encoding
is graphically shown here.
So this is an imaging sequence.
So you first have 90-degree pulse.
So just the 90-degree pulse makes the m vector
flip into xy plane.
So these transverse components will induce alternating field
around the patient or the small sample.
Then you can get a signal.
With the slice selection gradients,
you make sure you only flip spins
within the selected slice.
Spins outside the slice, we don't care.
So this slice selection does a trick removing one dimension.
This is one step forward.
Then you do phase encoding.

Why you have this multiple line?
You do phase gradients.
Once you do here, then you move it.
Or maybe you go from minimum.
Step by step, you need to do multiple phase encodings.
And for the reason, and you will see clear,
because that's a Fourier transformation,
you need a two-degree of freedom.
So each time, you have one phase encoding line.
And for each of multiple phase encoding line,
you need to do frequency encoding.
And the frequency encoding, you just get an n data point out.
So this is n data point along the horizontal axis.
And these multiple phase encoding lines
will cause the bounding to multiple data
points along vertical direction.
So this may be n by n.
Just map the whole cross section into n by n image.
So this is the Fourier imaging mode
or case-based formulation.
So if you call this whole thing for frequency encoding,
gx times t, this whole thing, call it kx.
And the phase encoding part, call it ky.
And then the previous formula you show
can be put into this format.
You see, the signal is proportional to this kx, x, ky, y.
So this is a Fourier transformation.
For any given time point, so you fix the time point,
you have a given kx.
And this ky determined by gy and tau phase encoding.
So you've got a phase encoding factor here,
frequency encoding factor here.
The frequency sampling is done if you like the time t
continuously goes, so it traces one line.
But for phase encoding, for given phase encoding line,
you only got one value here.
But for two-dimensional Fourier transformation,
you need to have multiple ky.
That's why I mentioned to you, you
have multiple horizontal phase encoding gradients
applied previously.
So this is the Fourier imaging formulation.
So the Fourier transformation, as defined before,
appears naturally as an equation 4.49.
So this is the key point.
And you see, this is a very important formulation.
And you may recall the Fourier transformation.
And what we learned is that they have 2 pi here.
Your textbook, 2 pi are missing.
And again, any textbook, including mine and the lecture,
is not very precise.
You really need to think through carefully.
So these 2 pi are missing.
You need to put it back.
So just small details here.
But the essential idea is through phase encoding
and the frequency encoding, you can measure data point.
Those data are naturally Fourier components.
Those data point immediately the Fourier information.
When we learn X-ray imaging and you have Fourier information
through application of Fourier slice theorem,
so the projection profile needs to be one-dimensionally
transformed to have a radial line in the Fourier space.
So you need some processing.

But for MRI imaging, the signal you measured
are immediately in the k-space.
So this is very important.
So if you do measurement, as we say,
so you have a phase factor.
And then you have the phase part appear as a y component.
And then the frequency encoding component
contributed as a kx.
And then you can go from different phase lines.
And then you let time go.
So the t equal to 0, you got 0 part.
And then say you get this one.
t keep increasing, you move just towards right-hand side.
You cover half space of Fourier domain.
And for real function, Fourier transform
has a good symmetric property.
If you just get half plane information,
then you can infer the other half.
So you can perform inverse Fourier transform
to get image reconstructed.
So this is the essential idea.
How can you collect data in k-space or Fourier space?
The k component is a Fourier component.
We call it k-space because now we're talking about space.
And then the Fourier domain, in the time space,
you have just a regular frequency.
And here you have a k-space.
You can also modify the pulse sequence a little bit.
You can have full coverage in k-space.
So what you would do is here.
So for frequency encoding, this is a frequency encoding.
Then you start data collection.
So before regular, the previously explained frequency
encoding and the data acquisition,
you just do a dephasing gradient
before this Fourier, before this frequency encoding.
Because this dephase encoding, so all the components,
the frequency components here, will
start with a negative vector.
If you don't do anything, you see here,
this kx times t, the measurement
goes from t equal to 0.
So you have a non-negative component.
But if you add a phase factor here,
so the kx component starts with a negative 1.
So you make it negative enough so that then you
can cover rectangular region, centralize that system origin.
And then the frequency range, like a Nyquist sampling
theorem, we think we have maximum bandwidth.
We want to make sure this overall rectangular region is
wide enough so effective Fourier components are
within the central region.
They are covered by this sampling range.
So the whole Fourier space, or k-space,
can be covered this way.
So this is what we need to know.
So how you use the k-space theorem to solve MRI problem.
And I have explained the key ideas to you,
assuming the phase encoding is along vertical direction,
frequency encoding along horizontal direction.
And the next slide, let me just explain things more generally.
So the tau is the time you spend.
This green tau is time you spend for phase encoding.
The phase encoding, you make a phase different,

but you really didn't collect the signal yet.
Then you let time t go.
The t is right.
And the right means you really start collecting signal.
This is the formulation with phase and frequency encoding.
So we mentioned that we first do phase encoding longitudinally.
Then we do frequency encoding just horizontally.
And then what if we do it differently?
So you can do it any way you want.
So when you do phase encoding, you
can turn vertical and horizontal gradient up.
So this way, given time tau, you move phase vector both ways
horizontally and vertically.
So you can put phase here.
So if you collect the signal immediately,
it will come from this point.
Or you can just turn the phase vector up,
move it to this point, then turn the frequency encoding.
You collect the signal.
You move on this way.
So this is just about what we explained.
So you have x gradient for frequency encoding.
That time t go.
So this kx will vary from t equal to 0
all the way towards positive.
You got this one.
So the other way, so you can first
use the horizontal gradient to do phase encoding.
You move the point initial state here.
Then you turn vertical gradient up.
You do phase encoding.
So this way, you just collect the case-based data
along this horizontal vertical line.
You just move this way.
The right line means you start collecting data.
Or you just do not do any phase encoding.
You turn vertical horizontal gradient on both time.
Then you collect the signal immediately.
Then no phase delay.
You start from system origin.
It moves along this straight line.
And the slope depends on relative magnitude
of vertical gradient and the horizontal gradient.
So the essential idea really, you
collect the signal with the gradient on,
the frequency will be different.
If you don't collect the signal, just turn gradient
on along certain direction over a certain amount of time,
the phase effect will be accumulated.
You can play any way you want.
So you can really explore the case-based or Fourier space
any way you want.
So this is just a rough idea.
So I'm not sure you totally follow me
or got confused a little bit.
I assure you, if you never read, never studied this before,
you must be confused.
As I said, for myself, I just studied very carefully
several times.
Then I got to understand clearly.
So just tell me, if you feel you basically
follow the logical flow, now raise your hand.
So I see you basically follow.
You got confused, quite confused.

You raise your hand and just see.
So if you didn't give me any signal,
I think you're kind of a little confused.
So this is what I expected.
That's why I say, let me explain again
with more detail from a different angle.
So let's just review the whole idea
to see if you get a little more better understanding.
So spin echo is a very clever but highly simplified signal
model.
So this is the illustration.
You have an n-vector, 90 degrees,
flip it into xy-plane.
In this case, along y-axis.
But you can also flip this way along x-axis.
It depends on where you apply the decimal field
to make this flipping.
But don't do anything, then it will deface, shown here.
Face delay, so face hide.
At this point, you got the magic 180 degree pulse.
You flip it the other way.
So this face delay will keep delay.
This face advance will keep moving faster.
And for the same amount of time, it will refacing.
Here, echo formed.
And this echo is from the selected slice.
Suppose you already did the slice selection.
So this is your equation 4.43.
So signal from a whole plane.
Signal from a whole plane.
And the signal from a whole plane
can be detected by a nearby coil.
Because this precisional vector, the transverse
component, will introduce an alternating EM field.
Alternating EM field will induce current in a nearby coil.
You learned in high school, the alternating field in the coil
will generate a signal here.
The signal will decay.
Why decay?
Because T2 star decay, it will keep decay.
And this signal is called a free induction decay, FID.
So this is signal observed from the whole sample.
After slice selection, it's from a whole slice,
otherwise from a whole sample.
You see this picture again.
This is the spin echo idea.
So you get a signal from a whole sample.
With slice selection, you get a signal
from the selected slice.
With phase encoding, with frequency encoding,
or readout, frequency encoding is for readout.
So you get a signal.
So this echo time, you get a signal out.
This signal reflects the Fourier transform
of the slice you selected.
Why you can have Fourier transformation?
Because you did phase encoding and the frequency encoding.
So you get spatially resolved information.
And I said that you can collect a signal with a nearby coil.
And then you put another coil.
You can also get a signal.
Remember, this horizontal component
is a rotational vector.
So the signal has two components.

So it depends on amplitude and the phase.
So the signal detected along x-axis and along y-axis,
they naturally have a 90-degree phase change.
Because this keeps changing.
It's a complex-valued quantity.
So you can record Sx modulate as M0.
So you use a 90-degree pulse.
And this is subject to your spin echo, subject to T2 decay.
So this is the amplitude.
And this is the frequency.
And likewise, you have Sy component.
It's just the sine, cosine.
So you have two components.
And you put it into complex form.
So you have a real part and an imaginary part.
You have a very compact, complex-valued signal shown here.
So I happen to put an imaginary part here corresponding
to signal Sy.
The coding system, Sy are arbitrarily chosen.
And for this one, I just make a Sy imaginary part.
But for physical measurement purposes, real part,
imaginary part, they are physically equally meaningful.
So just for mathematical notation,
I put in compact form.
As I explained Fourier series, Fourier transform,
I also mentioned that this idea using complex form.
And the notation becomes very compact.
And the signal for real function, the signal are real.
The complex form can always be converted into real form.
Like a Fourier transformation.
I have a real signal, the Fourier transformation.
And the inverse Fourier transform, all real.
But you can use a complex notation.
So now you have this complex notation.
And the precisional frequency is omega 0 determined by b0.
So keep doing this way at the angular frequency omega 0.
And if we use a rotating frame, so we rotate at the same
frequency, we will see a stationary vector there.
So in rotating frame, we just remove this.
The notation becomes even simpler.
We always want to make simplification.
If we really understand something,
then the key ideas are always simple and not very
complicated.
If you have to explain long time about some idea,
and with high likelihood, you really don't understand it.
So we always try to make it straightforward.
So this is the signal in rotating frame.
The signal is stationary without this omega 0
precisional rotation.
But this signal amplitude still decay, subject to T2 decay.
This is a physiological thing.
You cannot do much about it.
And therefore, signal, spin echo signal,
we say we repeat the whole thing after TR time repetition.
So if TR is not long enough, the vertical vector m0 is not,
the original vector m0 is not totally recovered yet.
T1 decay determines how large the amplitude of m0
could be recovered after TR.
So this is repeated after TR, you got this signal.
Then the signal whatsoever recovered,
and you use 90 degree pulse, you flip it.
The signal still subject to decay.
At time t, the overall signal will

be combination of this TR, the T1 recovery and the T2 defacing.
So this is a total signal determined by proton density,
proton density already hidden in the m0.
The higher proton density, the larger m0.
Then also T1, this vector, how much
you recovered after TR and the T2.
So these are biological.
And about the patient status, we want to know.
And the B0, TR, TET, all these are technical.
You can select.
Does not reflect patient pathology.
So this is signal model for 90 degree pulse.
And why you need a repetition?
Because we say for phase encoding,
each time you've got one line, then one level.
Then use frequency encoding, you sample the case phase
along one line.
The case phase is two dimensional.
You need to make multiple phase lines.
So you need to repeat.
So for tomography imaging, pretty much
like helical scanning.
And the third generation fan beam scanning,
you need to do scanning.
Here, you do case phase imaging kind of MRI scanning.
So you need to repeat.
You cannot collect all the data once for all.
If the TR is too long, the M0 will be fully recovered.
So that's neat.
That's clean.
So after M0 is fully recovered, you do 90 degree flipping.
So signal will be quite large.
If the TR is too short, maybe some M0 components
hasn't been fully recovered.
The signal will be compromised a little bit.
But a little bit smaller TR means the scanning time
will be shorter.
So the scanning speed is also very important.
You need to have all these combined together,
make image quality the best for the intended application.
Then the TR time for repetition and the TE time for echo
can be selected.
They are technical factors.
By your selection, you are sensing different things,
the different TR, TE combination.
And the signal will be a little different.
So this is, I think, also a tricky slice.
And you can use short TR or long TR.
You can use short TE or long TE.
And with different combinations, your image
can reflect more about rho, about T1, or T2.
So different combinations, you measure different parameters.
So this is not like CT, just the mu.
You have T1, T2, and rho.
If you just do one measurement, one kind of image,
the image really reflects all of these parameters.
But if you purposely emphasize one of the three parameters,
you need to select the proper technical parameters.
So first, let me explain.
What do I mean by long TR?
TR is the time for repetition.
If you wait long enough, both of them, you do.
You did with the m vector.
The long enough time, the m vector

will return to the original position,
which is along z-axis.
So m0 aligned with z-axis.
So long TR means m0 already returned
to the original location along z-axis.
And short TR means you do not give the system enough time
so that the full value of m0 has not
been recovered along z-axis.
So that's what we mean by long or short TR.
Short TE time for echo.
What do we mean by short TE?
Short TE means echo time, the spin echo.
The time you measure the echo, TE, is very short.
So the flip m vector doesn't have time to deface.
Because the echo, you flip, then you wait a short time,
you get echo.
So the defacing is not serious.
So mxy means in-plane components,
transverse components after 90 degree flipping
has a little time to deface.
So no T2 effect.
If you measure signal, you cannot tell how much is T2.
Because T2 doesn't have time to show its effect.
So if you use long TR, use long TR, long enough,
the m vector already recovered.
You have no information about T1.
Just wait long enough.
The T1 effect, either T1 is large or small.
If you wait long enough, just the m0
return to the original position.
So if you use short TR, here is short TR, and short TE.
Short TE means you cannot see T2 effect.
Short TR, the TR is not long enough.
So this recovery along the z-axis is not reached yet.
So this is called T1 weighted.
The T1 effect will show, because T1 really governs the process.
You return to the original location along z-axis.
If you do it soon enough, not fully recovered,
depends on the degree of the longitudinal component
recovery, you can see what is T1.
So likewise, it's a green arrow.
So if you use long TE, the echo time is long.
So the T2 decay will be reflected.
And also, the TR is long.
Make sure TR is long.
The original m0 is fully recovered.
So it's fully recovered.
So you do not have a T1 effect that plays a role.
So this is T2 weighted.
It's a green arrow.
If you use long TR for no T1 effect,
short TE, no T2 effect, then the signal
really proportional to rho.
The original m vector will be measured.
And no T1 involved, no T2 involved.
So it is shown as these three spots.
And this is not possible.
This means the short TR, long TE.
Earlier, I mentioned TR is repetition.
And the echo is reading this repetition time.
So the TE normally shouldn't be longer than TR.
So this is not possible.
So three cases shown here.
Again, this takes time for you to understand.

But if you still feel confused, if this is the first time,
that's natural.
So just think more, and you will understand.
So I'll give you some example about brain images.
On the same slide, you can do rho, not P-weighted.
It's rho-weighted.
And T1-weighted, T2-weighted.
In brain, you have mainly three, five things.
Green matter and white matter, cerebrospinal fluid, CSF.
You have blood, you have water.
You see T1, T2 are quite different.
So the same slide depends on how you do MRI imaging.
You can emphasize rho, T1, or T2 for different purposes.
And for special case, you use GX grid encoding for frequency
differentiation.
You have the overall signal.
So due to proton density, due to T1, T2.
So this is the overall thing.
This is amplitude of a local signal.
Then you use GX for frequency encoding.
The signal here, just to summarize what we discussed.
We know that signal detected can be put in complex form.
So you have this signal.
So this is frequency of a local signal.
So this shows us function, the frequency component.
It's a frequency component.
And for case-based data acquisition,
and this is the function.
This is a transformation kernel.
So you call it KX.
So KX will be traced along the time trajectory.
So you make a T increase, and you keep getting signal
in case-based, just along horizontal direction.
In this case, because we used the gridding field, the GX.
So this is a special case.
So you trace one line here.
But for a more general case, you can
apply a general gridding field.
Could be along any direction.
Then you have this, you have this.
Not sure if you can read this.
The gridding field, you have arbitrary XYZ gridding field.
Then you can move along X for a certain amount,
along Y a certain amount, along G a certain amount.
So after you move along XYZ respectively,
so you really have a 3D point in the 3D space.
So the local field will be determined by inner product.
So we read this K. Inside this, really GX times X
is X component.
GY times Y is Y component.
So how all these things together,
you define the KX as the K of T. As this gridding,
the local field or local frequency
is inner product of general G vector and position vector.
You do this time integral.
There's a total phase factor.
Put a K of T here.
So you've got this part, this inner product.
You look at this, this I for XYZ,
determined by this whole thing, is a function
you are going to recover.
And this is a Fourier transformation kernel.
So this is something you can control.
You move any direction you want.

Really depends on how you define GX, GY, ZZ.
And all these components can be time-varying.
So you can go anywhere you want.
So this is a general form of a case-based theorem.
Again, let me say it again, the GX, GY, ZZ, the gridding field
can be time-varying.
So you can just move in the line for a certain way.
Then you change GX, GY, ZZ.
You move to another way.
And this is a very neat Fourier space formulation.
But this formulation, I say, is only approximate.
Where are the approximations?
And it really is three places.
The first thing, you say, here is a function of T.
But when you talk about a case-based theorem,
you think this is a whole thing.
It's F of XYZ is not time-varying.
But actually, you have a time vector here.
And you ignore this.
So this is an approximation.
Second, you see this R. You think it's a positional vector.
It's fixed.
But the spin may move, like the spin in blood
will keep moving.
The patient may move.
So you really ignore the patient motion.
You think there is some physiological motion,
molecular motion.
You think this is a constant.
Also, this factor, you measure it.
You see the signal.
The signal really also subject to T2 decay.
But you just think you read out fast enough.
No T2 start decay yet.
So under all these conditions, you
have Fourier transformation.
But if you want to have a very accurate imaging,
you need all these factors.
But once you introduce these factors,
it's no longer Fourier transformation.
The reconstruction will become not easy to solve.
And we can use machine learning to solve the problem.
And maybe in the last lecture, I will say a little bit.
So here is an example.
Under the approximations, we can measure the Fourier space
information.
This only shows you the amplitude.
The Fourier transformation is a complex function.
You have phase and amplitude.
We only show amplitude something like this.
If you perform inverse Fourier transformation,
so you've got this part.
You've got the cross-sectional image recovered.
So I think this is an amazing achievement.
So with MRI, totally different from X3CT.
You've got a cross-sectional image.
So the last part is MRI scanner.
I wouldn't explain.
Just these are some slides.
You can read the textbook.
It's easy to read.
It's just descriptive.
Just read it, you'll understand.
Very, very interesting.

Very interesting knowledge.
And a few links, if you want to understand the spin echo,
gradient echo, and the selection.
And it's not required.
It's just a decimal material.
And the homework.